



## Problem Solving for Irish Second level Mathematicians

Thursday, 15th October 2015

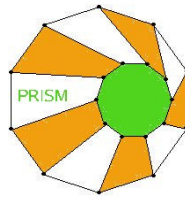
### *PRISM PROBLEMS*

Time allowed: 60 minutes

#### Rules and Guidelines for Contestants

1. You are **not** allowed to use a calculator or any measuring device (e.g. ruler or protractor).
2. **Use a pencil to fill out the answer sheet.** If you make a mistake, you can erase the error and correct it.
3. Write your name clearly (in block capitals) in the space provided in the answer sheet.
4. You should have some extra sheets of your own paper (or a refill pad) for rough work while you are doing the questions.
5. When you have decided on your answer for a particular question, carefully mark your choice for that question on the answer sheet.
6. Do not make any other marks on the answer sheet other than to write your name and to mark your answers to the questions.
7. Some of the questions are quite difficult, (especially for junior students who will find the latter part of the paper to be challenging) and we do not expect that many people will have time to think about all of them in 60 minutes. You will probably do better if you concentrate on a few rather than trying to guess the answer to all of the questions. The questions at the beginning are easier than those at the end. The problems are meant to encourage you to think! Don't be in a rush to mark your answer to any of the questions - take your time, read the questions carefully and make sure you understand what is being asked before you start to figure out the answer.
8. **There is no pass/fail mark in PRISM.** Correct answers will score one point each; incorrect or omitted answers will score zero.

*Good luck and thank you for participating in PRISM.  
We hope you will enjoy the problems!*



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*The PRISM Problems for 2015*

- What is the next number in the sequence 64, 32, 16, 8, 4?  
(A) 1 (B) 2 (C) 3 (D) 4 (E) None of these
- Which of the following numbers is not divisible by 3?  
(A) 9 (B) 18 (C) 28 (D) 36 (E) 45
- Andrew has 4 toy cars, Liam has 6 toy cars and Elizabeth has  $n$  toy cars. If each of Andrew and Liam gave one half of their cars to Elizabeth, she would then have twice the number of cars that Andrew and Liam would have between them. What is  $n$ ?  
(A) 1 (B) 3 (C) 5 (D) 6 (E) 10
- Tom is 2 years older than Mary. If the sum of their ages is 20, what is the product of their ages?  
(A) 99 (B) 100 (C) 102 (D) 110 (E) None of these
- Adam paid €1.00 for 2 chocolate bars and €2.10 for 3 cakes. If the price of chocolate bars was *increased* by 10% and the price of cakes *decreased* by 10%, how much would he pay for 3 chocolate bars and 2 cakes?  
(A) €2.91 (B) €3.00 (C) €3.11 (D) €3.20 (E) €3.20
- Consider the following four statements (1),(2),(3) and (4).  
(1)  $2^2 + 2^3 = 2^5$  (2)  $4(3(2 - 1) - 2) = 4$  (3)  $\frac{1}{3} + \frac{1}{3} = \frac{3}{2}$  (4)  $\frac{1}{20} + \frac{1}{21} < \frac{1}{10}$   
Then it is the case that  
(A) None of statements (1)–(4) is true  
(B) Exactly one of statements (1)–(4) is true  
(C) Exactly two of statements (1)–(4) are true  
(D) Exactly three of statements (1)–(4) are true  
(E) All four of statements (1)–(4) are true
- Which of the following numbers is the greatest?  
(A) 1 (B)  $\frac{3}{2}$  (C)  $\sqrt{2}$  (D)  $\sqrt[3]{3}$  (E)  $\sqrt[4]{4}$
- Of the 200 students in a class, 100 were male, 100 studied Art, and 60 were neither male nor studied Art. How many of the 200 students were male and studied Art?  
(A) 100 (B) 80 (C) 60 (D) 50 (E) 40

9. What is the measure (in degrees) of the angle between two adjacent sides of a regular pentagon (i.e. a shape with 5 equal sides and 5 equal angles)?

- (A)  $60^\circ$  (B)  $72^\circ$  (C)  $108^\circ$  (D)  $120^\circ$  (E)  $144^\circ$

10. Peter has 4 blue socks and 10 black socks in a drawer. He is also completely colour blind. What is the smallest number of socks that he must pack to ensure that he has at least one pair of blue socks and at least one pair of black socks in his suitcase?

- (A) 4 (B) 6 (C) 8 (D) 10 (E) 12

11. What is the correct ranking of the numbers  $3^{200}$ ,  $64^{50}$  and  $7^{100}$  in order from smallest to largest?

- (A)  $3^{200} < 64^{50} < 7^{100}$   
(B)  $3^{200} < 7^{100} < 64^{50}$   
(C)  $7^{100} < 64^{50} < 3^{200}$   
(D)  $64^{50} < 7^{100} < 3^{200}$   
(E)  $64^{50} < 3^{200} < 7^{100}$

12. Which of the following has the greatest volume?

- (A) A cube with edges of length 2  
(B) A cube with diagonal of length 4 (the diagonal is the length from one corner of the cube to the corner of the cube that is furthest away from it)  
(C) A cube with surface area 20  
(D) A cube with volume 7  
(E) A cube, each of whose faces has a diagonal of length 3

13. A car was driven up a hill at a constant speed of 30km/h. At what speed was it driven down the same hill if the average speed of the car for the entire journey was 40km/h?

- (A) 35km/h  
(B) 45km/h  
(C)  $50\frac{1}{3}$ km/h  
(D) 60km/h  
(E) Impossible to say - we need to know the distance from the bottom to the top of the hill

14. If  $xy = 5$  and  $x - y = 10$  then  $x^2 + y^2$  is equal to

- (A) 15 (B) 55 (C) 75 (D) 100 (E) 110

15. If a boat takes 2 hours to travel down a river (with the current) from point A to point B, and 3 hours to travel up the river (against the current) from B to A, how many hours would it take the boat to travel from A to B in still water? (Assume that the boat always travels at the same speed *relative to the water*.)

- (A)  $\frac{3}{2}$  (B)  $\frac{5}{2}$  (C)  $\frac{12}{5}$  (D)  $\frac{24}{5}$  (E) More information is required

16. How many integers between 1 and 1,000 inclusive are divisible by at least one of the two numbers 4 and 6?

- (A) 500    (B) 480    (C) 444    (D) 333    (E) 240

17. Which of the following numbers is not divisible by 9?

(A) 9

(B)  $3^{100} - 27$

(C) 12 345 678 987 654 321

(D)  $9^8 7^6 5^4 3^2 1$

(E) 222 222 277 777 771

18. A farmer brings a load of watermelons to the market. Before he sets out, he measures the total mass and the percentage water content. He finds that the total mass is 100kg and the water content is 99%. (So there is 1kg of solids and there is 99kg of water.) The weather is hot, so his load loses some moisture en route. He checks the water content when he arrives at the market: it has dropped to 98%. What is the total mass of the load on arrival at market?

- (A) 1kg    (B) 2kg    (C) 50kg    (D) 98kg    (E) 99kg

19. Sean works in Dublin and flies home by plane to Cork every evening. He arrives at the airport in Cork at exactly 9 pm each evening. His wife always meets the plane and drives Sean to their house. One evening however, Sean took an earlier plane and arrived at the airport in Cork at 8 pm. He started walking along the route always taken by his wife. They meet somewhere on the road, Sean hopped into the car, and they arrived at their house 10 minutes earlier than usual. If Sean's wife drove at a constant speed and left just in time to meet the 9 pm plane, for how long did Sean walk before he was picked up?

- (A) 5 minutes    (B) 10 minutes    (C) 30 minutes    (D) 50 minutes    (E) 55 minutes

20. Antonia, Dara and Tosia are identical triplet sisters in the same class. Antonia always tells the truth, Dara always lies and Tosia sometimes lies and sometimes tells the truth. One day one of them arrives late for class. The teacher asks this late sister who she is. She answers 'I am Tosia'. The teacher cannot tell the girls apart so asks the other two sisters the name of the sister who was late. One of them says: 'Antonia was late', and the other says: 'Dara was late'. Which sister was in fact late?

(A) Antonia

(B) Dara

(C) Tosia

(D) Either Dara or Tosia

(E) It could be any of the three