**Problem Solving for Irish Second level Mathematicians**
**Thursday, 16th October 2014**

***Senior Level***

**Time allowed: 60 minutes**

**Rules and Guidelines for Contestants**

1. You are **not** allowed to use a calculator or any measuring device (e.g. ruler or protractor).
2. **Use a pencil to fill out the answer sheet**. If you make a mistake, you can erase the error and correct it.
3. Write your name clearly (in block capitals) in the space provided in the answer sheet.
4. You should have some extra sheets of your own paper (or a refill pad) for rough work while you are doing the questions.
5. When you have decided on your answer for a particular question, carefully mark your choice for that question on the answer sheet.
6. Do not make any other marks on the answer sheet other than to write your name and to mark your answers to the questions.
7. Some of the questions are quite difficult, and we do not expect that many people will have time to think about all of them in 60 minutes. You will probably do better if you concentrate on a few rather than trying to guess the answer to all of the questions.
   The questions at the beginning are generally easier than those at the end.
   The problems are meant to encourage you to think! Don't be in a rush to mark your answer to any of the questions - take your time, read the questions carefully and make sure you understand what is being asked before you start to figure out the answer.
8. **There is no pass/fail mark in PRISM.** Correct answers will score one point each; incorrect or omitted answers will score zero.

*Good luck and thank you for participating in PRISM.*
*We hope you will enjoy the problems!*

1. What is the smallest positive number that is divisible by each of 3, 7 and 14?

   **(A)** 14 **(B)** 21 **(C)** 42 **(D)** 98 **(E)** 294

2. What are the solutions of the equation $x^2 + x - 12 = 0$?

   **(A)** 0,12 **(B)** 3,4 **(C)** $-3,-4$ **(D)** $-3,4$ **(E)** $3,-4$

3. Let $a$ be any negative integer. Which of the following numbers has the greatest value?

   **(A)** $-3a$ **(B)** $6a$ **(C)** $-a+1$ **(D)** $a$ **(E)** $a-2$

4. The mean mark of five students in an exam was 60. If four of the students have their marks raised by 6 and one student has his mark lowered by 4, what now will the mean mark of the five students be?

   **(A)** 62 **(B)** 63 **(C)** 64 **(D)** 65 **(E)** 66

5. If the letters of the word MATHS are arranged in all possible ways, what is the probability that a random arrangement will commence with the letter M or the letter A?

   **(A)** 0.0167 **(B)** 0.1 **(C)** 0.2 **(D)** 0.3 **(E)** 0.4

6. What is the remainder when $(x^2 - x - 1)^{40}$ is divided by $x - 2$?

   **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

7. What is the correct ranking of the numbers $9^{\frac{3}{2}}$, $4^{\frac{5}{2}}$ and 30 in order from smallest to largest?

   **(A)** $9^{\frac{3}{2}} < 4^{\frac{5}{2}} < 30$

   **(B)** $9^{\frac{3}{2}} < 30 < 4^{\frac{5}{2}}$

   **(C)** $4^{\frac{5}{2}} < 9^{\frac{3}{2}} < 30$

   **(D)** $30 < 9^{\frac{3}{2}} < 4^{\frac{5}{2}}$

   **(E)** $30 < 4^{\frac{5}{2}} < 9^{\frac{3}{2}}$

8. John looked at the picture of a man on a wall and said: "Brothers and sisters I have none but that man's father is my father's son". What is the relationship between the man in the picture and John?

   **(A)** The man is the picture is John's father

   **(B)** The man is the picture is John's son

   **(C)** The man is the picture is John

   **(D)** The man is the picture is John's grandfather

   **(E)** There is insufficient information given to determine the relationship between the two men

9. What is the value of $\frac{(3i)^{10}+3^{10}}{3^{10}}$ where $i^2 = -1$?

**(A)** $0$ **(B)** $1$ **(C)** $2$ **(D)** $i+1$ **(E)** $i-1$

10. If the function $f(x)$ has roots $x = 3$ and $x = 5$, what are the solutions to the equation $f(x-2) = 0$?

**(A)** $5, 7$ **(B)** $5, 5$ **(C)** $2, 3$ **(D)** $1, \frac{3}{5}$ **(E)** none of these

11. In how many ways can 6 identical apples be distributed among 4 children if each child must get at least one apple?

**(A)** $1$ **(B)** $4$ **(C)** $5$ **(D)** $10$ **(E)** $15$

12. In a survey of 100 secondary school teachers, 20 said they teach mathematics. Among these 20 mathematics teachers, 12 were females. If a report of Irish secondary schools stated "A survey of 100 secondary school teachers showed that 60% of them are female teachers of mathematics", which of the following statements is valid based on the above information?

**(A)** The report is accurate because 12 out of 20 teachers is 60%

**(B)** The report is accurate because $(100 - 20 - 20)$ out of 100 teachers is 60%

**(C)** The report may be accurate because many male teachers also teach mathematics

**(D)** The report is inaccurate because only 12% of the 100 teachers are female teachers of mathematics

**(E)** The report is inaccurate because only 30% of the 100 teachers are female teachers of mathematics

13. Assume that the probability of a child being born a boy is $\frac{1}{2}$. Suppose a man says, "I have two children; at least one of them is a boy". What is the probability that the other one is a boy?

**(A)** $1$ **(B)** $\frac{1}{2}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{4}$ **(E)** $0$

14. Asia wrote down a number of consecutive members of the set $\{0, 1, 2, ..., 9\}$. How many of the five fractions $\frac{2}{5}$, $\frac{1}{2}$, $\frac{4}{7}$, $\frac{3}{5}$, $\frac{5}{8}$ could represent the proportion of even numbers in her list?

**(A)** $1$ **(B)** $2$ **(C)** $3$ **(D)** $4$ **(E)** $5$

15. A triangle has one side of length 6cm and a second side of length 3cm. If the angle between these two sides is $60°$, what is the length of the third side in cm? Note if you need it that $cos60° = sin30° = \frac{1}{2}$

**(A)** $9$ **(B)** $\sqrt{4}5$ **(C)** $\sqrt{2}7$ **(D)** $\sqrt{2}1$ **(E)** none of these

16. The vertices of the triangle $ABC$ are the points $A(1, 2), B(4, 6)$and $C(-4, 12)$. Which one of the following statements about triangle $ABC$ is true?

**(A)** $ABC$ is a right-angled triangle with the right angle at $A$

**(B)** $ABC$ is a right-angled triangle with the right angle at $B$

**(C)** $ABC$ is a right-angled triangle with the right angle at $C$

**(D)** $ABC$ is not a right-angled triangle but is an equilateral triangle

**(E)** $ABC$ is not a right-angled triangle and is not an equilateral triangle

17. The digits of the number 1234 can be arranged in all possible ways to produce 24 numbers $1234, 1243, 1324, 1342, \ldots, 4312, 4321$. What is the sum of these 24 four-digit numbers?

**(A)** 4936 **(B)** 14808 **(C)** 29516 **(D)** 66660 **(E)** 88880

18. If a boat takes three days to travel downstream from A to B and four days to come back upstream, how many days would it take a wooden log to be carried from A to B by the current? Assume the speed of the current is constant.

**(A)** 36 **(B)** 24 **(C)** 12 **(D)** 3.5 **(E)** 3

19. A home heating oil tank has three pumps A, B and C. Pumps A and B are used to pump oil into the tank, and a much smaller pump C is used to pump oil from the tank into the home (each pump works at its own constant rate). When pump A alone is in operation, an empty tank can be filled by it in 10 minutes. When pump B alone is in operation, an empty tank can be filled in 20 minutes. When pump C alone is in operation, a full tank takes 18,000 minutes to empty. This morning the tank was empty and the three pumps were turned on simultaneously (pumps A and B pumping oil into the tank and pump C pumping oil out). To the nearest minute, how long did it take for the tank to fill up?

**(A)** 3 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 11

20. How many zeros are there at the end of the number $30! = 30 \times 29 \times 28 \times ... \times 3 \times 2 \times 1$?

**(A)** 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 10